Data Warehousing
資料倉儲

Data Cube Computation and Data Generation

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<th>Topic</th>
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<td>12</td>
<td>100/05/03</td>
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<td>100/05/10</td>
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<td>15</td>
<td>100/05/24</td>
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Data Cube Computation and Data Generalization

• **Efficient Computation of Data Cubes**
• **Exploration and Discovery in Multidimensional Databases**
• **Attribute-Oriented Induction — An Alternative Data Generalization Method**
Efficient Computation of Data Cubes

- Preliminary cube computation tricks (Agarwal et al.’96)
- Computing full/iceberg cubes: 3 methodologies
  - Top-Down: Multi-Way array aggregation (Zhao, Deshpande & Naughton, SIGMOD’97)
  - Bottom-Up:
    - Bottom-up computation: BUC (Beyer & Ramarkrishnan, SIGMOD’99)
    - H-cubing technique (Han, Pei, Dong & Wang: SIGMOD’01)
  - Integrating Top-Down and Bottom-Up:
    - Star-cubing algorithm (Xin, Han, Li & Wah: VLDB’03)
- High-dimensional OLAP: A Minimal Cubing Approach (Li, et al. VLDB’04)
- Computing alternative kinds of cubes:
  - Partial cube, closed cube, approximate cube, etc.
Preliminary Tricks (Agarwal et al. VLDB’96)

• Sorting, hashing, and grouping operations are applied to the dimension attributes in order to reorder and cluster related tuples

• Aggregates may be computed from previously computed aggregates, rather than from the base fact table
  – Smallest-child: computing a cuboid from the smallest, previously computed cuboid
  – Cache-results: caching results of a cuboid from which other cuboids are computed to reduce disk I/Os
  – Amortize-scans: computing as many as possible cuboids at the same time to amortize disk reads
  – Share Sorts: sharing sorting costs cross multiple cuboids when sort-based method is used
  – Share-partitions: sharing the partitioning cost across multiple cuboids when hash-based algorithms are used
Multi-Way Array Aggregation

- Array-based “bottom-up” algorithm
- Using multi-dimensional chunks
- No direct tuple comparisons
- Simultaneous aggregation on multiple dimensions
- Intermediate aggregate values are reused for computing ancestor cuboids
- Cannot do Apriori pruning: No iceberg optimization
Multi-way Array Aggregation for Cube Computation (MOLAP)

- Partition arrays into chunks (a small subcube which fits in memory).
- Compressed sparse array addressing: (chunk_id, offset)
- Compute aggregates in “multiway” by visiting cube cells in the order which minimizes the # of times to visit each cell, and reduces memory access and storage cost.

What is the best traversing order to do multi-way aggregation?
Multi-way Array Aggregation for Cube Computation
Multi-way Array Aggregation for Cube Computation
Multi-Way Array Aggregation for Cube Computation (Cont.)

• Method: the planes should be sorted and computed according to their size in ascending order
  – Idea: keep the smallest plane in the main memory, fetch and compute only one chunk at a time for the largest plane

• Limitation of the method: computing well only for a small number of dimensions
  – If there are a large number of dimensions, “top-down” computation and iceberg cube computation methods can be explored
Bottom-Up Computation (BUC)

- BUC (Beyer & Ramakrishnan, SIGMOD’99)
- Bottom-up cube computation (Note: top-down in our view!)
- Divides dimensions into partitions and facilitates iceberg pruning
  - If a partition does not satisfy \( min\_sup \), its descendants can be pruned
  - If \( minsup = 1 \) \( \Rightarrow \) compute full CUBE!
- No simultaneous aggregation
BUC: Partitioning

- Usually, entire data set can’t fit in main memory
- Sort *distinct* values, partition into blocks that fit
- Continue processing
- Optimizations
  - Partitioning
    - External Sorting, Hashing, Counting Sort
  - Ordering dimensions to encourage pruning
    - Cardinality, Skew, Correlation
  - Collapsing duplicates
    - Can’t do holistic aggregates anymore!
Star-Cubing: An Integrating Method

• Integrate the top-down and bottom-up methods

• **Explore shared dimensions**
  – E.g., dimension A is the shared dimension of ACD and AD
  – ABD/AB means cuboid ABD has shared dimensions AB

• Allows for shared computations
  – e.g., cuboid AB is computed simultaneously as ABD

• Aggregate in a top-down manner but with the bottom-up sub-layer underneath which will allow Apriori pruning

• Shared dimensions grow in bottom-up fashion
Iceberg Pruning in Shared Dimensions

• Anti-monotonic property of shared dimensions
  – If the measure is anti-monotonic, and if the aggregate value on a shared dimension does not satisfy the iceberg condition, then all the cells extended from this shared dimension cannot satisfy the condition either

• Intuition: if we can compute the shared dimensions before the actual cuboid, we can use them to do Apriori pruning

• Problem: how to prune while still aggregate simultaneously on multiple dimensions?
Cell Trees

- Use a tree structure similar to H-tree to represent cuboids
- Collapses common prefixes to save memory
- Keep count at node
- Traverse the tree to retrieve a particular tuple
Star Attributes and Star Nodes

- Intuition: If a single-dimensional aggregate on an attribute value \( p \) does not satisfy the iceberg condition, it is useless to distinguish them during the iceberg computation
  - E.g., \( b_2, b_3, b_4, c_1, c_2, c_4, d_1, d_2, d_3 \)
- Solution: Replace such attributes by a *. Such attributes are *star attributes*, and the corresponding nodes in the cell tree are *star nodes*
Example: Star Reduction

• Suppose minsup = 2
• Perform one-dimensional aggregation. Replace attribute values whose count < 2 with *. And collapse all *’s together
• Resulting table has all such attributes replaced with the star-attribute
• With regards to the iceberg computation, this new table is a loseless compression of the original table

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>a1</td>
<td>b1</td>
<td>*</td>
<td>*</td>
<td>1</td>
</tr>
<tr>
<td>a1</td>
<td>b1</td>
<td>*</td>
<td>*</td>
<td>1</td>
</tr>
<tr>
<td>a1</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>1</td>
</tr>
<tr>
<td>a2</td>
<td>*</td>
<td>c3</td>
<td>d4</td>
<td>1</td>
</tr>
<tr>
<td>a2</td>
<td>*</td>
<td>c3</td>
<td>d4</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>a1</td>
<td>b1</td>
<td>*</td>
<td>*</td>
<td>2</td>
</tr>
<tr>
<td>a1</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>1</td>
</tr>
<tr>
<td>a2</td>
<td>*</td>
<td>c3</td>
<td>d4</td>
<td>2</td>
</tr>
</tbody>
</table>
• Efficient Computation of Data Cubes

• Exploration and Discovery in Multidimensional Databases

• Attribute-Oriented Induction — An Alternative Data Generalization Method
Computing Cubes with Non-Antimonotonic Iceberg Conditions

• Most cubing algorithms cannot compute cubes with non-antimonotonic iceberg conditions efficiently

• Example

CREATE CUBE Sales_Iceberg AS
SELECT month, city, cust_grp,
    AVG(price), COUNT(*)
FROM Sales_Infor
CUBE BY month, city, cust_grp
HAVING AVG(price) >= 800 AND COUNT(*) >= 50

• Needs to study how to push constraint into the cubing process
Non-Anti-Monotonic Iceberg Condition

• Anti-monotonic: if a process fails a condition, continue processing will still fail
• The cubing query with avg is non-anti-monotonic!
  – (Mar, *, *, 600, 1800) fails the HAVING clause
  – (Mar, *, Bus, 1300, 360) passes the clause

<table>
<thead>
<tr>
<th>Month</th>
<th>City</th>
<th>Cust_grp</th>
<th>Prod</th>
<th>Cost</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>Tor</td>
<td>Edu</td>
<td>Printer</td>
<td>500</td>
<td>485</td>
</tr>
<tr>
<td>Jan</td>
<td>Tor</td>
<td>Hld</td>
<td>TV</td>
<td>800</td>
<td>1200</td>
</tr>
<tr>
<td>Jan</td>
<td>Tor</td>
<td>Edu</td>
<td>Camera</td>
<td>1160</td>
<td>1280</td>
</tr>
<tr>
<td>Feb</td>
<td>Mon</td>
<td>Bus</td>
<td>Laptop</td>
<td>1500</td>
<td>2500</td>
</tr>
<tr>
<td>Mar</td>
<td>Van</td>
<td>Edu</td>
<td>HD</td>
<td>540</td>
<td>520</td>
</tr>
<tr>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
</tr>
</tbody>
</table>

CREATE CUBE Sales_Iceberg AS
SELECT month, city, cust_grp, AVG(price), COUNT(*)
FROM Sales_Infor
CUBE BY month, city, cust_grp
HAVING AVG(price) >= 800 AND COUNT(*) >= 50
From Average to Top-k Average

- Let \((*, \text{Van}, *)\) cover 1,000 records
  - \(\text{Avg}(\text{price})\) is the average price of those 1000 sales
  - \(\text{Avg}^{50}(\text{price})\) is the average price of the top-50 sales (top-50 according to the sales price)

- Top-k average is anti-monotonic
  - The top 50 sales in Van. is with \(\text{avg}(\text{price}) \leq 800\) → the top 50 deals in Van. during Feb. must be with \(\text{avg}(\text{price}) \leq 800\)
Binning for Top-k Average

• Computing top-k avg is costly with large k
• Binning idea
  – $\text{Avg}^{50}(c) \geq 800$
  – Large value collapsing: use a sum and a count to summarize records with measure $\geq 800$
    • If count $\geq 800$, no need to check “small” records
  – Small value binning: a group of bins
    • One bin covers a range, e.g., 600~800, 400~600, etc.
    • Register a sum and a count for each bin
Computing Approximate top-k average

Suppose for (\(*, \text{Van}, *\)), we have

<table>
<thead>
<tr>
<th>Range</th>
<th>Sum</th>
<th>Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Over 800</td>
<td>28000</td>
<td>20</td>
</tr>
<tr>
<td>600~800</td>
<td>10600</td>
<td>15</td>
</tr>
<tr>
<td>400~600</td>
<td>15200</td>
<td>30</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

Approximate avg\(^{50}\)() = 
\[
\frac{28000+10600+600\times15}{50}=952
\]

Top 50

The cell may pass the HAVING clause

<table>
<thead>
<tr>
<th>Month</th>
<th>City</th>
<th>Cust_grp</th>
<th>Prod</th>
<th>Cost</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>
Weakened Conditions Facilitate Pushing

- Accumulate quant-info for cells to compute average iceberg cubes efficiently
  - Three pieces: sum, count, top-k bins
  - Use top-k bins to estimate/prune descendants
  - Use sum and count to consolidate current cell

Approximate $\text{avg}^{50}()$
- Anti-monotonic, can be computed efficiently

Real $\text{avg}^{50}()$
- Anti-monotonic, but computationally costly

$\text{avg}()$
- Not anti-monotonic
Computing Iceberg Cubes with Other Complex Measures

• Computing other complex measures
  – Key point: find a function which is weaker but ensures certain anti-monotonicity

• Examples
  – Avg() \leq v: \text{avg}_k(c) \leq v \ (\text{bottom-k avg})
  – Avg() \geq v \ (\text{no count}): \max(\text{price}) \geq v
  – Sum(profit) \ (\text{profit can be negative}):
    • p\_sum(c) \geq v \text{ if } p\_count(c) \geq k; \text{ or otherwise, } \text{sum}^k(c) \geq v
  – Others: conjunctions of multiple conditions
Compressed Cubes: Condensed or Closed Cubes


- Icerberg cube cannot solve all the problems
  - Suppose 100 dimensions, only 1 base cell with count = 10. How many aggregate (non-base) cells if count >= 10?

- Condensed cube
  - Only need to store one cell \((a_1, a_2, \ldots, a_{100}, 10)\), which represents all the corresponding aggregate cells
  - Adv.
    - Fully precomputed cube without compression
    - Efficient computation of the minimal condensed cube

- Closed cube
  - Dong Xin, Jiawei Han, Zheng Shao, and Hongyan Liu, “C-Cubing: Efficient Computation of Closed Cubes by Aggregation-Based Checking”, ICDE'06.
• Efficient Computation of Data Cubes

• Exploration and Discovery in Multidimensional Databases

• Attribute-Oriented Induction — An Alternative Data Generalization Method
Discovery-Driven Exploration of Data Cubes

• Hypothesis-driven
  – exploration by user, huge search space

• Discovery-driven (Sarawagi, et al.’98)
  – Effective navigation of large OLAP data cubes
  – pre-compute measures indicating exceptions, guide user in the data analysis, at all levels of aggregation
  – Exception: significantly different from the value anticipated, based on a statistical model
  – Visual cues such as background color are used to reflect the degree of exception of each cell
Kinds of Exceptions and their Computation

• Parameters
  – SelfExp: surprise of cell relative to other cells at same level of aggregation
  – InExp: surprise beneath the cell
  – PathExp: surprise beneath cell for each drill-down path

• Computation of exception indicator (modeling fitting and computing SelfExp, InExp, and PathExp values) can be overlapped with cube construction

• Exception themselves can be stored, indexed and retrieved like precomputed aggregates
## Examples: Discovery-Driven Data Cubes

### Sum of sales

<table>
<thead>
<tr>
<th>Sum of sales</th>
<th>month</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Jan</td>
</tr>
<tr>
<td>Total</td>
<td>1%</td>
</tr>
</tbody>
</table>

### Avg sales

<table>
<thead>
<tr>
<th>item</th>
<th>month</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Jan</td>
</tr>
<tr>
<td>Sony b/w printer</td>
<td>9%</td>
</tr>
<tr>
<td>Sony color printer</td>
<td>0%</td>
</tr>
<tr>
<td>HP b/w printer</td>
<td>-2%</td>
</tr>
<tr>
<td>HP color printer</td>
<td>0%</td>
</tr>
<tr>
<td>IBM home computer</td>
<td>1%</td>
</tr>
<tr>
<td>IBM laptop computer</td>
<td>0%</td>
</tr>
<tr>
<td>Toshiba home computer</td>
<td>-2%</td>
</tr>
<tr>
<td>Toshiba laptop computer</td>
<td>1%</td>
</tr>
<tr>
<td>Logitech mouse</td>
<td>3%</td>
</tr>
<tr>
<td>Ergo-way mouse</td>
<td>0%</td>
</tr>
</tbody>
</table>

### IBM home computer

<table>
<thead>
<tr>
<th>Avg sales</th>
<th>month</th>
</tr>
</thead>
<tbody>
<tr>
<td>region</td>
<td>Jan</td>
</tr>
<tr>
<td>North</td>
<td>-1%</td>
</tr>
<tr>
<td>South</td>
<td>-1%</td>
</tr>
<tr>
<td>East</td>
<td>-1%</td>
</tr>
<tr>
<td>West</td>
<td>4%</td>
</tr>
</tbody>
</table>
Complex Aggregation at Multiple Granularities: Multi-Feature Cubes


- Example: Grouping by all subsets of \{item, region, month\}, find the maximum price in 1997 for each group, and the total sales among all maximum price tuples.

  ```sql
  select item, region, month, max(price), sum(R.sales)
  from purchases
  where year = 1997
  cube by item, region, month: R
  such that R.price = max(price)
  ```

- Continuing the last example, among the max price tuples, find the min and max shelf life, and find the fraction of the total sales due to tuples that have min shelf life within the set of all max price tuples.
Cube-Gradient (Cubegrade)

- Analysis of changes of sophisticated measures in multi-dimensional spaces
  - Query: changes of average house price in Vancouver in ‘00 comparing against ’99
  - Answer: Apts in West went down 20%, houses in Metrotown went up 10%
- Cubegrade problem by Imielinski et al.
  - Changes in dimensions $\rightarrow$ changes in measures
  - Drill-down, roll-up, and mutation
From Cubegrade to Multi-dimensional Constrained Gradients in Data Cubes

• Significantly more expressive than association rules
  – Capture trends in user-specified measures

• Serious challenges
  – Many trivial cells in a cube → “significance constraint” to prune trivial cells
  – Numerate pairs of cells → “probe constraint” to select a subset of cells to examine
  – Only interesting changes wanted → “gradient constraint” to capture significant changes
MD Constrained Gradient Mining

- Significance constraint $C_{sig}$: (cnt $\geq$ 100)
- Probe constraint $C_{prb}$: (city = “Van”, cust_grp = “busi”, prod_grp = “*”)
- Gradient constraint $C_{grad}(c_g, c_p)$: 
  \[
  \frac{\text{avg}\_\text{price}(c_g)}{\text{avg}\_\text{price}(c_p)} \geq 1.3
  \]

<table>
<thead>
<tr>
<th>Base cell</th>
<th>Aggregated cell</th>
<th>Siblings</th>
<th>Ancestor</th>
</tr>
</thead>
<tbody>
<tr>
<td>(c4, c2) satisfies $C_{grad}$!</td>
<td>(c4, c2) satisfies $C_{grad}$!</td>
<td>(c4, c2) satisfies $C_{grad}$!</td>
<td>(c4, c2) satisfies $C_{grad}$!</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dimensions</th>
<th>Measures</th>
</tr>
</thead>
<tbody>
<tr>
<td>cid</td>
<td>Yr</td>
</tr>
<tr>
<td>c1</td>
<td>00</td>
</tr>
<tr>
<td>c2</td>
<td>*</td>
</tr>
<tr>
<td>c3</td>
<td>*</td>
</tr>
<tr>
<td>c4</td>
<td>*</td>
</tr>
</tbody>
</table>
Efficient Computing Cube-gradients

• Compute probe cells using $C_{\text{sig}}$ and $C_{\text{prb}}$
  – The set of probe cells $P$ is often very small

• Use probe $P$ and constraints to find gradients
  – Pushing selection deeply
  – Set-oriented processing for probe cells
  – Iceberg growing from low to high dimensionalities
  – Dynamic pruning probe cells during growth
  – Incorporating efficient iceberg cubing method
• Efficient Computation of Data Cubes

• Exploration and Discovery in Multidimensional Databases

• Attribute-Oriented Induction — An Alternative Data Generalization Method
What is Concept Description?

- Descriptive vs. predictive data mining
  - **Descriptive mining**: describes concepts or task-relevant data sets in concise, summarative, informative, discriminative forms
  - **Predictive mining**: Based on data and analysis, constructs models for the database, and predicts the trend and properties of unknown data

- Concept description:
  - **Characterization**: provides a concise and succinct summarization of the given collection of data
  - **Comparison**: provides descriptions comparing two or more collections of data
Data Generalization and Summarization-based Characterization

• Data generalization
  – A process which abstracts a large set of task-relevant data in a database from a low conceptual levels to higher ones.

  – Approaches:
    • Data cube approach (OLAP approach)
    • Attribute-oriented induction approach
Concept Description vs. OLAP

• Similarity:
  – Data generalization
  – Presentation of data summarization at multiple levels of abstraction.
  – Interactive drilling, pivoting, slicing and dicing.

• Differences:
  – Can handle complex data types of the attributes and their aggregations
  – Automated desired level allocation.
  – Dimension relevance analysis and ranking when there are many relevant dimensions.
  – Sophisticated typing on dimensions and measures.
  – Analytical characterization: data dispersion analysis
Attribute-Oriented Induction

• Proposed in 1989 (KDD ‘89 workshop)
• Not confined to categorical data nor particular measures
• How it is done?
  – Collect the task-relevant data (*initial relation*) using a relational database query
  – Perform generalization by *attribute removal* or *attribute generalization*
  – Apply aggregation by merging identical, generalized tuples and accumulating their respective counts
  – Interactive presentation with users
Basic Principles of Attribute-Oriented Induction

- **Data focusing**: task-relevant data, including dimensions, and the result is the *initial relation*
- **Attribute-removal**: remove attribute $A$ if there is a large set of distinct values for $A$ but (1) there is no generalization operator on $A$, or (2) $A$’s higher level concepts are expressed in terms of other attributes
- **Attribute-generalization**: If there is a large set of distinct values for $A$, and there exists a set of generalization operators on $A$, then select an operator and generalize $A$
- **Attribute-threshold control**: typical 2-8, specified/default
- **Generalized relation threshold control**: control the final relation/rule size
Attribute-Oriented Induction: Basic Algorithm

- **InitialRel**: Query processing of task-relevant data, deriving the *initial relation*.
- **PreGen**: Based on the analysis of the number of distinct values in each attribute, determine generalization plan for each attribute: removal? or how high to generalize?
- **PrimeGen**: Based on the PreGen plan, perform generalization to the right level to derive a “prime generalized relation”, accumulating the counts.
- **Presentation**: User interaction: (1) adjust levels by drilling, (2) pivoting, (3) mapping into rules, cross tabs, visualization presentations.
Example

- **DMQL**: Describe general characteristics of graduate students in the Big-University database
  
  ```
  use Big_University_DB
  mine characteristics as "Science_Students"
  in relevance to name, gender, major, birth_place, birth_date, residence, phone#, gpa
  from student
  where status in "graduate"
  ```

- **Corresponding SQL statement**:  
  ```
  Select name, gender, major, birth_place, birth_date, residence, phone#, gpa
  from student
  where status in {"Msc", "MBA", "PhD" }
  ```
## Class Characterization: An Example

<table>
<thead>
<tr>
<th>Name</th>
<th>Gender</th>
<th>Major</th>
<th>Birth-Place</th>
<th>Birth_date</th>
<th>Residence</th>
<th>Phone #</th>
<th>GPA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jim Woodman</td>
<td>M</td>
<td>CS</td>
<td>Vancouver, BC, Canada</td>
<td>8-12-76</td>
<td>3511 Main St., Richmond</td>
<td>687-4598</td>
<td>3.67</td>
</tr>
<tr>
<td>Scott Lachance</td>
<td>M</td>
<td>CS</td>
<td>Montreal, Que, Canada</td>
<td>28-7-75</td>
<td>345 1st Ave., Richmond</td>
<td>253-9106</td>
<td>3.70</td>
</tr>
<tr>
<td>Laura Lee</td>
<td>F</td>
<td>Physics</td>
<td>Seattle, WA, USA</td>
<td>25-8-70</td>
<td>125 Austin Ave., Burnaby</td>
<td>420-5232</td>
<td>3.83</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
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<td>...</td>
<td>...</td>
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</tr>
</tbody>
</table>

**Prime Generalized Relation**

<table>
<thead>
<tr>
<th>Gender</th>
<th>Major</th>
<th>Birth_region</th>
<th>Age_range</th>
<th>Residence</th>
<th>GPA</th>
<th>Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>Science</td>
<td>Canada</td>
<td>20-25</td>
<td>Richmond</td>
<td>Very-good</td>
<td>16</td>
</tr>
<tr>
<td>F</td>
<td>Science</td>
<td>Foreign</td>
<td>25-30</td>
<td>Burnaby</td>
<td>Excellent</td>
<td>22</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

### Birth Region

<table>
<thead>
<tr>
<th>Gender</th>
<th>Total</th>
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</thead>
<tbody>
<tr>
<td>M</td>
<td>30</td>
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<tr>
<td>F</td>
<td>32</td>
</tr>
<tr>
<td>Total</td>
<td>62</td>
</tr>
</tbody>
</table>

### Initial Relation

<table>
<thead>
<tr>
<th>Name</th>
<th>Gender</th>
<th>Major</th>
<th>Birth-Place</th>
<th>Birth_date</th>
<th>Residence</th>
<th>Phone #</th>
<th>GPA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jim Woodman</td>
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<td>CS</td>
<td>Vancouver, BC, Canada</td>
<td>8-12-76</td>
<td>3511 Main St., Richmond</td>
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</tr>
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<td>CS</td>
<td>Montreal, Que, Canada</td>
<td>28-7-75</td>
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<td>Laura Lee</td>
<td>F</td>
<td>Physics</td>
<td>Seattle, WA, USA</td>
<td>25-8-70</td>
<td>125 Austin Ave., Burnaby</td>
<td>420-5232</td>
<td>3.83</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td></td>
</tr>
</tbody>
</table>

### Removed

- Gender: M
- Major: Science
- Birth_region: Canada
- Age_range: 20-25
- Residence: Richmond
- GPA: Very-good
- Phone #: 687-4598
- Count: 16

### Retained

- Gender: F
- Major: Science
- Birth_region: Foreign
- Age_range: 25-30
- Residence: Burnaby
- GPA: Excellent
- Phone #: 420-5232
- Count: 22

### Excluded

- Gender: M
- Major: Science
- Birth_region: Canada
- Age_range: 20-25
- Residence: Richmond
- GPA: Very-good
- Phone #: 687-4598
- Count: 16

- Gender: F
- Major: Science
- Birth_region: Foreign
- Age_range: 25-30
- Residence: Burnaby
- GPA: Excellent
- Phone #: 420-5232
- Count: 22

### Total

- Gender: M
- Major: Science
- Birth_region: Canada
- Age_range: 20-25
- Residence: Richmond
- GPA: Very-good
- Phone #: 687-4598
- Count: 30

- Gender: F
- Major: Science
- Birth_region: Foreign
- Age_range: 25-30
- Residence: Burnaby
- GPA: Excellent
- Phone #: 420-5232
- Count: 32

- Total: 62
Presentation of Generalized Results

• **Generalized relation:**
  – Relations where some or all attributes are generalized, with counts or other aggregation values accumulated.

• **Cross tabulation:**
  – Mapping results into cross tabulation form (similar to contingency tables).
  – **Visualization techniques:**
  – Pie charts, bar charts, curves, cubes, and other visual forms.

• **Quantitative characteristic rules:**
  – Mapping generalized result into characteristic rules with quantitative information associated with it, e.g.,

\[
\text{grad}(x) \land \text{male}(x) \Rightarrow \\
\text{birth\_region}(x) = \text{"Canada"}[t:53\%] \lor \text{birth\_region}(x) = \text{"foreign"}[t:47\%].
\]
Mining Class Comparisons

• **Comparison**: Comparing two or more classes

• **Method**:
  – Partition the set of relevant data into the target class and the contrasting class(es)
  – Generalize both classes to the same high level concepts
  – Compare tuples with the same high level descriptions
  – Present for every tuple its description and two measures
    • support - distribution within single class
    • comparison - distribution between classes
  – Highlight the tuples with strong discriminant features

• **Relevance Analysis**:
  – Find attributes (features) which best distinguish different classes
Quantitative Discriminant Rules

- $C_j = \text{target class}$
- $q_a = \text{a generalized tuple covers some tuples of class}$
  - but can also cover some tuples of contrasting class
- $d$-weight
  - range: $[0, 1]$
  - \[ d\text{-weight} = \frac{\text{count}(q_a \in C_j)}{\sum_{i=1}^{m} \text{count}(q_a \in C_i)} \]
- quantitative discriminant rule form

\[ \forall X, \ target\_class(X) \iff \text{condition}(X) \ [d : d\_weight] \]
Example: Quantitative Discriminant Rule

<table>
<thead>
<tr>
<th>Status</th>
<th>Birth_country</th>
<th>Age_range</th>
<th>Gpa</th>
<th>Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Graduate</td>
<td>Canada</td>
<td>25-30</td>
<td>Good</td>
<td>90</td>
</tr>
<tr>
<td>Undergraduate</td>
<td>Canada</td>
<td>25-30</td>
<td>Good</td>
<td>210</td>
</tr>
</tbody>
</table>

Count distribution between graduate and undergraduate students for a generalized tuple

• Quantitative discriminant rule

∀X, graduate_student(X) ←

birth_country(X) ="Canada" ∧ age_range(X) ="25−30" ∧ gpa(X) ="good" [d : 30%]

− where 90/(90 + 210) = 30%
Class Description

• Quantitative characteristic rule

\[ \forall X, \ target\_class(X) \Rightarrow \text{condition}(X) \ [t : t\_weight] \]
– necessary

• Quantitative discriminant rule

\[ \forall X, \ target\_class(X) \Leftarrow \text{condition}(X) \ [d : d\_weight] \]
– sufficient

• Quantitative description rule

\[ \forall X, \ target\_class(X) \Leftrightarrow \]
\[ \text{condition}_1(X)[t : w_1, d : w'_1] \lor \ldots \lor \text{condition}_n(X)[t : w_n, d : w'_n] \]
– necessary and sufficient
### Example: Quantitative Description Rule

<table>
<thead>
<tr>
<th>Location/item</th>
<th>TV</th>
<th></th>
<th>Computer</th>
<th></th>
<th>Both_items</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Count</td>
<td>t-wt</td>
<td>d-wt</td>
<td>Count</td>
<td>t-wt</td>
<td>d-wt</td>
</tr>
<tr>
<td>Europe</td>
<td>80</td>
<td>25%</td>
<td>40%</td>
<td>240</td>
<td>75%</td>
<td>30%</td>
</tr>
<tr>
<td>N_Am</td>
<td>120</td>
<td>17.65%</td>
<td>60%</td>
<td>560</td>
<td>82.35%</td>
<td>70%</td>
</tr>
<tr>
<td>Both_regions</td>
<td>200</td>
<td>20%</td>
<td>100%</td>
<td>800</td>
<td>80%</td>
<td>100%</td>
</tr>
</tbody>
</table>

Crosstab showing associated t-weight, d-weight values and total number (in thousands) of TVs and computers sold at AllElectronics in 1998

- Quantitative description rule for target class *Europe*

\[ \forall X, Europe(X) \iff (item(X) = "TV") [t : 25\%, d : 40\%] \lor (item(X) = "computer") [t : 75\%, d : 30\%] \]
Summary

• Efficient algorithms for computing data cubes
• Further development of data cube technology
  – Discovery-drive cube
  – Multi-feature cubes
  – Cube-gradient analysis
• Another generalization approach: Attribute-Oriented Induction
References

• Jiawei Han and Micheline Kamber, Data Mining: Concepts and Techniques, Second Edition, 2006, Elsevier